

## Elementary Algebra Topics

Name \_\_\_\_\_

Elementary Algebra (17%) (10-11 questions). EXAMPLES taken from previous ACT practice tests

### Properties of Exponents and Square Roots

22. For all  $a > 1$ , the expression  $\frac{3a^4}{3a^6}$  equals:

F.  $\frac{1}{2}$

G.  $-a^2$

H.  $a^2$

J.  $-\frac{1}{a^2}$

K.  $\frac{1}{a^2}$

$$\frac{3a^4}{3a^6} = \frac{1}{a^{6-4}} = \frac{1}{a^2}$$

35.  $(3x^3)^3$  is equivalent to:

A.  $x$

B.  $9x^6$

C.  $9x^9$

D.  $27x^6$

E.  $27x^9$

$$(3x^3)^3 = 3^3 x^{3 \cdot 3} = 27x^9$$

36. Which of the following is equivalent to the inequality  $4x - 8 > 8x + 16$ ?

F.  $x < -6$

G.  $x > -6$

H.  $x < -2$

J.  $x > 2$

K.  $x < 6$

$$\begin{aligned} 4x - 8 &> 8x + 16 \\ 4x - 8x &> 16 + 8 \\ -4x &> 24 \\ \frac{-4x}{-4} &> \frac{24}{-4} \\ x &< -6 \end{aligned}$$

IF BOTH SIDES of an INEQUATIONS are multiplied or divided by the same negative number, the inequality sign is reversed

40. If there are  $8 \times 10^{12}$  hydrogen molecules in a volume of  $4 \times 10^4$  cubic centimeters, what is the average number of hydrogen molecules per cubic centimeter?

F.  $5 \times 10^{-9}$

G.  $2 \times 10^3$

H.  $2 \times 10^8$

J.  $32 \times 10^{16}$

K.  $32 \times 10^{48}$

$$\frac{8 \times 10^{12}}{4 \times 10^4} = 2 \times 10^{12-4} = 2 \times 10^8$$

45. Which of the following is a rational number?

Definitions

A.  $\sqrt{2}$

B.  $\sqrt{\pi}$

C.  $\sqrt{7}$

D.  $\sqrt{\frac{5}{25}}$

E.  $\sqrt{\frac{64}{49}}$

from  
"ratio"  
 $-\frac{1}{2}$  or  $\frac{1}{2}$   
 $\frac{3}{1}$  or  $3:1$

RATIONAL number can be made by dividing one integer (positive/negative whole number) by another  
Examples:  $\frac{1}{2}$ ,  $2 = \frac{2}{1}$ ,  $2.12 = \frac{212}{100}$

IRRATIONAL Number cannot be written as ratio or fraction  $\rightarrow \pi, \sqrt{2}$  or  $\sqrt{5}$

49. In the real numbers, what is the solution of the equation  $8^{2x+1} = 4^{1-x}$ ?

A.  $-\frac{1}{3}$

B.  $-\frac{1}{4}$

C.  $-\frac{1}{8}$

D. 0

E.  $\frac{1}{7}$

$8^{2x+1} = 4^{1-x}$  ← change to same base numbers

$(2^3)^{2x+1} = (2^2)^{1-x}$

$2^{6x+3} = 2^{2-2x}$  same base

$6x+3 = 2-2x$   
 $+2x -3 -3+2x$

$8x = -1$   
 $x = -\frac{1}{8}$

YOU TRY Properties of Exponents and Square Roots:

1. The value of  $(2.5 \cdot 10^5)^2$  is equal to which of the following?

F.  $6.25 \cdot 10^7$

G.  $6.25 \cdot 10^{10}$

H.  $2.5 \cdot 10^{10}$

J.  $2.7 \cdot 10^{10}$

K.  $5 \cdot 10^7$

$2.5^2 \cdot 10^{5 \times 2} = 6.25 \cdot 10^{10}$

2. If  $n^x \cdot n^8 = n^{24}$  and  $(n^6)^y = n^{18}$ , what is the value of  $x + y$ ?

A. 7

B. 9

C. 12

D. 19

E. 27

$n^{x+8} = n^{24}$  same base  $(n^6)^y = n^{18}$

$x+8=24$   
 $-8 -8$

$x=16$

$n^{6y} = n^{18}$

$6y=18$   
 $\frac{6y}{6} = \frac{18}{6}$

$y=3$

$x+y=16+3$   
 $=19$

3.  $\frac{10^3(10^5 + 10^5)}{10^4} = ?$

- F.  $10^4$   
 G.  $10^6$   
 H.  $2(10^2)$   
☒ J.  $2(10^4)$   
 K.  $2(10^9)$

$$\frac{10^3(2 \cdot 10^5)}{10^4 10^1} = \frac{2 \cdot 10^5}{10^1} = 2 \cdot 10^4$$

4.  $2^0 + 2^3 - 2^{-2} = ?$

- A. 4  
 B.  $6\frac{1}{4}$   
 C. 7  
☒ D.  $8\frac{3}{4}$   
 E.  $9\frac{3}{4}$

$$\begin{aligned} 2^3 &= 8 \\ 2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1 \\ 2^{-1} &= \frac{1}{2} \\ 2^{-2} &= \frac{1}{4} \end{aligned} \quad 2^0 + 2^3 - 2^{-2} = 1 + 8 - \frac{1}{4} = 8\frac{3}{4}$$

### Evaluation of Algebraic Expressions through Substitution

PEDMAS or BIDMAS

1. What is the value of the expression  $6x - [7 - 2(3x - 10)]$  when  $x = 5$ ?

- A. -27  
 B. -17  
 C. 13  
☒ D. 33  
 E. 87

$$\begin{aligned} &6x - [7 - 2(3x - 10)] \text{ when } x = 5 \\ &6(5) - [7 - 2(3 \cdot 5 - 10)] \\ &30 - [7 - 2(5)] \\ &30 - [7 - 10] = 30 - -3 = 30 + 3 = 33 \end{aligned}$$

2. When  $x = -2$ , what is the value of  $x^3 - x + 3$ ?

- A. -5  
☒ B. -3  
 C. -1  
 D. 9  
 E. 13

$$\begin{aligned} &(-2)^3 - (-2) + 3 \rightarrow (-2)(-2)(-2) \\ &-8 + 2 + 3 \quad 4(-2) \\ &-8 + 5 = -3 \quad -8 \end{aligned}$$

# Using variables to express functional relationships

2. A consultant charges \$45 for each hour she works on a consultation, plus a flat \$30 consulting fee. How many hours of work are included in a \$210 bill for a consultation?

F. 2

G. 4

H. 4

J. 5

K. 7

$$\begin{array}{r} 45h + 30 = 210 \\ -30 \quad -30 \\ \hline 45h = 180 \\ \underline{45} \quad \underline{45} \\ h = 180/45 = 4 \end{array}$$

24. The fixed costs of manufacturing basketballs in a factory are \$1,400.00 per day. The variable costs are \$5.25 per basketball. Which of the following expressions can be used to model the cost of manufacturing  $b$  basketballs in 1 day?

F. \$1,405.25 $b$

G. \$5.25 $b$  - \$1,400.00

H. \$1,400.00 $b$  + \$5.25

J. \$1,400.00 - \$5.25 $b$

K. \$1,400.00 + \$5.25 $b$

\$1400.00/day

\$1400.00 + \$5.25 $b$

basketball variable 5.25 $b$

27. A hot-air balloon 70 meters above the ground is falling at a constant rate of 6 meters per second while another hot-air balloon 10 meters above the ground is rising at a constant rate of 15 meters per second. To the nearest tenth of a second, after how many seconds will the 2 balloons be the same height above the ground?

A. 8.9

B. 6.7

C. 2.9

D. 0.4

E. 0.2

70 fall 6m/s

$$70 - 6s = 10 + 15s$$

$$70 - 10 = 15s + 6s$$

$$\frac{60}{21} = \frac{21s}{21}$$

$$2.857 = s$$

10 rise 15m/s

58. For every positive 2-digit number,  $x$ , with tens digit  $t$  and units digit  $u$ , let  $y$  be the 2-digit number formed by reversing the digits of  $x$ . Which of the following expressions is equivalent to  $x - y$ ?

F.  $9(t - u)$

G.  $9(u - t)$

H.  $9t - u$

J.  $9u - t$

K. 0

$$x = 10t + u$$

$$y = 10u + t$$

$$x - y = 10t + u - (10u + t)$$

$$x - y = 10t + u - 10u - t$$

$$x - y = 9t - 9u = 9(t - u)$$



**YOU TRY** Using variables to express functional relationships:

1. Guillermo earns take-home pay of \$6.75 per hour. Out of his next paycheck, he would like to buy 3 books for \$7.48 each and rent 4 video games for \$5.34 each (both prices include tax). Let  $n$  represent the number of hours for which Guillermo is paid on his next pay-check. Which of the following inequalities, when solved, gives the values of  $n$  that allow Guillermo to buy the books and rent the video games?

F.  $7.48 + 5.34 \leq n$

G.  $7.48 + 5.34 \leq 6.75n$

H.  $7(7.48 + 5.34) \leq 6.75n$

J.  $3(7.48) + 4(5.34) \leq n$

K.  $3(7.48) + 4(5.34) \leq 6.75n$

G earn  $6.75/\text{hr}$  books =  $3(\$7.48)$  vid =  $4(\$5.34)$

G buys  $3(7.48) + 4(5.34)$

$n$  = number of hours worked

$3(7.48) + 4(5.34) \leq 6.75n$

**Understanding Algebraic Operations**

4.  $t^2 - 59t + 54 - 82t^2 + 60t$  is equivalent to:

F.  $-26t^2$

G.  $-26t^6$

H.  $-81t^4 + t^2 + 54$

J.  $-81t^2 + t + 54$

K.  $-82t^2 + t + 54$

$t^2 - 59t + 54 - 82t^2 + 60t$

$t^2 - 82t^2 - 59t + 60t + 54$

$-81t^2 + t + 54$

6. The expression  $(4z + 3)(z - 2)$  is equivalent to:

F.  $4z^2 - 5$

G.  $4z^2 - 6$

H.  $4z^2 - 3z - 5$

J.  $4z^2 - 5z - 6$

K.  $4z^2 + 5z - 6$

F Multiply First terms

O Multiply Outer terms

I Multiply Inner terms

L Multiply Last terms

$(4z+3)(z-2)$

$4z^2 - 8z + 3z - 6$

$4z^2 - 5z - 6$

8. The 6 consecutive integers below add up to 447.

$x - 2$

$x - 1$

$x$

$x + 1$

$x + 2$

$x + 3$

$x - 2 + x - 1 + x + x + 1 + x + 2 + x + 3 = 447$

$6x - 3 + 6 = 6x + 3 = 447$

$-3 \quad -3$

$6x = 444$

$x = 74$

What is the value of  $x$ ?

F. 72

G. 73

H. 74

J. 75

K. 76

- F.  $a + b$   
G.  $-(a + b)$   
H.  $\sqrt{a - b}$   
J.  $a - b$   
K.  $-(a - b)$

$|a - b|$  is equivalent to which of the

$a < b$  example #1  $1 < 2$  for  $|a - b|$  then  $|1 - 2|$

example  $9 < 10 \rightarrow |9 - 10| = |-1|$

$| -1 | = 1$

$| -3 | = 3$

$| -1 | = 1$

$| -3 | = 3$

absolute value - always positive

1.  $(x + 2)(x - 4) - (x + 4)(x - 2) = ?$

1.  $(x+2)(x-4) - (x+4)(x-2) = ?$   
FOIL  $[x^2 - 4x + 2x - 8] - [x^2 - 2x + 4x - 8] = x^2 - 2x - 8 - (x^2 + 2x - 8) = x^2 - 2x - 8 - x^2 - 2x + 8 = -4x$   
E.O.

- F. 0  
G.  $2x^2 + 4x - 16$   
**H.  $-4x$**   
J.  $4x$   
K.  $-4x - 16$

- A.  $5x - 5$   
B.  $5x - 4$   
C.  $5x - 1$   
D.  $x - 5$   
E.  $-5x$

- F. 6  
G. 5  
H. 4  
J. 3  
K. 2

$$\begin{array}{r} -2x + 5 = 2 - (5 - 2x) \\ -2x + 5 = 2 - 5 + 2x \\ +2x \qquad \qquad +2x \\ 5 = -3 + 4x \\ +3 \quad +3 \\ \frac{8}{4} = \frac{4x}{4} \qquad 2 = x \end{array}$$

- F. -7  
G. -5  
H. -3  
J. 5  
K. 7

$$\begin{array}{r} 5y + 3 = 2y - 18 \\ -2y \quad -2y \\ 3y + 3 = -18 \\ -3 \quad -3 \\ 3y = -21 \\ y = -7 \end{array}$$

5. For 2 numbers,  $c$  and  $d$ ,  $c$  is 4 less than the product of 3 and the number  $d$ . Which of the following is an expression for  $c$ , in terms of  $d$ ?

- A.  $3(d+4)$   
 B.  $3(d-4)$   
 C.  $4-3d$   
 D.  $3d+4$   
 E.  $3d-4$

$$c = 3d - 4$$

6. Which of the following expressions correctly describes the mathematical relationship below?

3 less than the product of 4 times  $x$

- F.  $4x-3$   
 G.  $3x-4$   
 H.  $4(x-3)$   
 J.  $3(4x)$   
 K.  $\frac{4x}{3}$

7. If  $\frac{3}{4}$  of  $x$  is 36, then  $\frac{1}{3}$  of  $x = ?$

- A. 9  
 B. 12  
 C. 16  
 D. 24  
 E. 42

$$\frac{3}{4}x = 36$$

$$\frac{4}{3} \times \frac{3}{4}x = \frac{36}{1} \times \frac{4}{3}$$

$$x = 48$$

$$\frac{1}{3}x = ?$$

$$\frac{1}{3}(48) = ?$$

$$16 =$$

8. If  $12 + x = 36 - y$ , then  $x + y = ?$

- F. -48  
 G. -24  
 H. 3  
 J. 24  
 K. 48

$$12 + x = 36 - y$$

$$-12 \quad -12$$

$$x = 24 - y$$

$$x + y = 24$$

9. For a positive integer  $k$ , which of the following equals  $6k + 3$ ?

- F.  $\frac{1}{2}(k+1)$   
 G.  $\frac{1}{k} + 4$   
 H.  $2k + 1$   
 J.  $3(k+1)$   
 K.  $3(2k+1)$

$$6k + 3$$

$$3(2k + 1)$$

# Solution of quadratic equations by factoring

21. What values of  $x$  are solutions for  $x^2 + 2x = 8$ ?

- A. -4 and 2
- B. -2 and 0
- C. -2 and 4
- D. 0 and 2
- E. 6 and 8

$$\begin{aligned} x^2 + 2x &= 8 \\ x^2 + 2x - 8 &= 0 \\ (x+4)(x-2) &= 0 \\ x+4=0 & \quad x-2=0 \\ x=-4 & \quad x=2 \end{aligned}$$

Factors of -8 then add  
 $-8 \times 1$  or  $8 \times -1$   
 $-4 \times 2$  or  $4 \times -2$

$$\begin{aligned} -8+1 &= -7 \\ 8-1 &= 7 \\ -4+2 &= -2 \\ 4-2 &= 2 \end{aligned}$$

## YOU TRY Solution of quadratic equations by factoring:

1. Which of the following is an element of the solution set of the equation  $x^2 + 6x + 8 = 0$ ?

- A. -8
- B. -2
- C. 4
- D. 6
- E. 8

$$\begin{aligned} x^2 + 6x + 8 &= 0 \\ (x+4)(x+2) &= 0 \\ x+4=0 & \quad x+2=0 \\ x=-4 & \quad x=-2 \end{aligned}$$

Factors of 8

$-8 \times -1$  or  $8 \times 1$   
 $-4 \times -2$  or  $4 \times 2$

then add

$$\begin{aligned} -8+1 &= -7 \\ 8+1 &= 9 \\ -4+2 &= -2 \\ 4+2 &= 6 \end{aligned}$$

2. If 1 of the roots of the equation  $x^2 + kx - 12 = 0$  is 4, what is the value of  $k$ ?

- F. -1
- G. 0
- H. 1
- J. 3
- K. 7

$$\begin{aligned} x^2 + kx - 12 &= 0 \\ x^2 + -1x - 12 &= 0 \\ (x-4)(x+3) &= 0 \end{aligned}$$

$$\begin{aligned} x+4=0 & \quad \text{NOT} \\ x=-4 & \quad \text{root 4} \\ x-4=0 & \\ x=4 & \end{aligned}$$

Factors of -12 with -4

$$\begin{aligned} -4 \times 3 & \xrightarrow{\text{ADD}} -4+3 = -1 \\ & \quad \uparrow \\ & \quad k \end{aligned}$$

3. Which of the following represents  $-7t + 6t^2 - 3$  when it is completely factored?

- A.  $(3t-1)(2t+3)$
- B.  $(3t+1)(2t-3)$
- C.  $(6t-1)(t+3)$
- D.  $(6t+1)(t-3)$
- E.  $(2t-1)(3t+3)$

$$\begin{aligned} -7t + 6t^2 - 3 &= 6t^2 - 7t - 3 = 0 \\ (3t+1)(2t-3) & \end{aligned}$$

Factors

$$\begin{aligned} 6 & \begin{cases} 6, 1 \\ -3, -2 \\ 2, 3 \end{cases} \quad 3 & \begin{cases} -3, 1 \\ 3, -1 \end{cases} \end{aligned}$$

6	-3
6	1
1	6
3	2
2	3
-3	-2
-2	-3

4. What is the solution set for the following equation:  $x^2 - 5x + 4 = 0$ ?

- A.  $\{-4, -1\}$
- B.  $\{-3, -1\}$
- C.  $\{-1, 3\}$
- D.  $\{1, 4\}$
- E.  $\{2, 3\}$

$$\begin{aligned} x^2 - 5x + 4 &= 0 \\ (x-4)(x-1) &= 0 \\ x-4=0 & \quad x-1=0 \\ +4 & \quad +4 \quad +1 \quad +1 \\ x=4 & \quad x=1 \end{aligned}$$

4
-4
4
-2
2